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## The Generation of Side Force by Distributed Suction

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## **Abstract**

This report provides an approximate analysis of the generation of side force on a cylinder placed horizontal to the flow direction by the application of distributed suction on the rearward side of the cylinder. Relationships are derived between the side force coefficients and the required suction coefficients necessary to maintain attached flow on one side of the cylinder, thereby inducing circulation around the cylinder and a corresponding side force.

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## **Table of Contents**

Abstract .....	2
Nomenclature .....	5
Introduction .....	6
Analysis.....	7
Potential Flow .....	7
Side Force Coefficient .....	8
Boundary Layer Separation .....	8
Required Normal Suction .....	11
Conclusion.....	15
References .....	16

## **Table of Figures**

Figure 1. Flow Over Cylinder .....	7
Figure 2. Pressure Side Separation Angle vs. Circulation Angle .....	10
Figure 3. Separation Angle with Suction vs. Circulation Angle .....	10
Figure 4. Side Force Coefficient vs. Circulation Angle .....	10
Figure 5. Suction Side Separation Angle vs. Circulation Angle .....	11
Figure 6. Suction Velocity Distribution.....	13
Figure 7. Suction Coefficient vs. Side Force Coefficient.....	14

## **Nomenclature**

$C_f$	Friction coefficient = $\tau_o / \frac{1}{2} \rho U^2$
$C_p$	Pressure coefficient = $(P - P_\infty) / \frac{1}{2} \rho U_\infty^2$
$C_s$	Suction coefficient = $\frac{1}{2} \pi \int_{\phi_2}^{\phi_1} v_o d\phi \left( \frac{Re}{10^6} \right)^{-\frac{1}{2}}$
$C_y$	Side force coefficient = $F_y / \frac{1}{2} \rho U_\infty^2 R$
$F_y$	Side force
$P$	Pressure
$R$	Radius of cylinder
$Re$	Reynolds number = $UR/\nu$
$U$	Velocity
$u$	Dimensionless velocity = $U/U_\infty$
$v_o$	Suction velocity at wall
$\delta^*$	Dimensionless displacement thickness = $\delta^*/R$
$\bar{\delta}^*$	Displacement thickness
$\phi$	Angle around cylinder
$\phi_o$	Circulation angle
$\phi_1$	Pressure side separation angle
$\phi_2$	Separation point due to suction
$\phi_s$	Suction side separation angle
$\Gamma$	Shape factor
$\theta$	Dimensionless momentum thickness = $\bar{\theta}/R$
$\bar{\theta}$	Momentum thickness
$\rho$	Density
$\tau$	Shear stress
$\tau_o$	Shear stress at wall

## **Introduction**

It is well known that the application of suction symmetrical to the rear of a cylinder placed horizontally in a flow can be used to delay flow separation and thereby reduce the drag of a cylinder. If suction is applied asymmetrically, separation will be delayed on that side and will produce an asymmetric outer flow corresponding to a circulation around the cylinder.

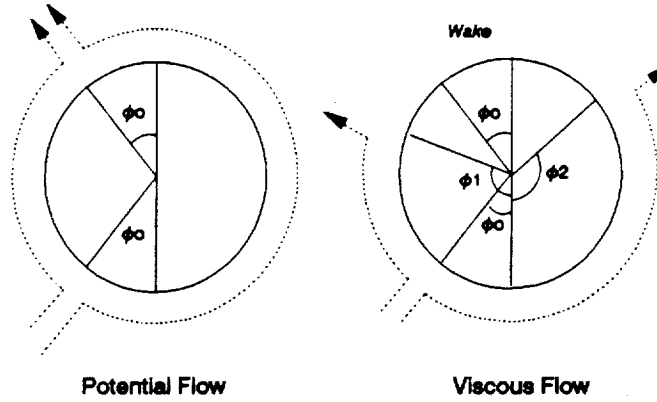
In this paper, distributed suction for both laminar and turbulent flow are considered. It is expected that the results will have application to the design of suction control devices for use on the fuselage of very high angle of attack fighter aircraft.





## Analysis

The flow over a cylinder with suction is illustrated in figure 1.



**Figure 1. Flow Over Cylinder**

The outer flow is considered to be that of a cylinder with circulation in which the forward stagnation point on the cylinder is displaced by an angle  $\varphi_0$  in the clockwise direction. The flow is considered to separate at the location  $\varphi_1$  on the pressure side of the cylinder and at an angle  $\varphi_2$  on the normal suction side. The normal suction distribution is then defined to be consistent with this separation geometry. In this way the required suction can be related directly to the circulation or the side force coefficient.

### **Potential Flow**

The velocity at the surface of the cylinder in the presence of circulation is

$$U = U_\infty u(\varphi)$$

$$\text{where } u(\varphi) = 2(\sin \varphi + \sin \varphi_0) \quad (1)$$

corresponding to a flow with a forward stagnation point at  $\varphi = -\varphi_0$  on the cylinder.

The pressure coefficient can be evaluated as

$$C_p = \frac{P - P_\infty}{\frac{1}{2} \rho U_\infty^2} = 1 - u^2(\varphi) = 1 - 4(\sin \varphi + \sin \varphi_0)^2 \quad (2)$$

This expression is considered to hold for the attached flow  $\varphi_1 < \varphi < -\varphi_0$  and  $-\varphi_0 < \varphi < \varphi_2$ . For the rearward part of the cylinder where the flow is separated,  $\varphi_1 < \varphi < \varphi_2$ , it is assumed that a base pressure  $C_p = C_{pB}$  applies. Furthermore, it is assumed that  $C_p(\varphi_1) = C_p(\varphi_2)$  which leads to a relationship between  $\varphi_1$  and  $\varphi_2$

$$\sin \varphi_1 + \sin \varphi_0 = -(\sin \varphi_2 + \sin \varphi_0)$$

$$\text{i.e. } \sin \varphi_2 = -(\sin \varphi_1 + 2 \sin \varphi_0) \quad (3)$$

Since  $\varphi_1$  is a function of  $\varphi_0$ <sup>1</sup> equation (3) gives  $\varphi_2$  as a function of  $\varphi_0$ .

### Side Force Coefficient

A side force coefficient based on the force per unit length of the cylinder is defined as

$$C_Y = \frac{F_Y}{\frac{1}{2} \rho U^2 R} = \int_0^{2\pi} C_p(\varphi) \sin \varphi d\varphi$$

$$= \int_{\varphi_1}^{\varphi_2} C_p(\varphi) \sin \varphi d\varphi + \int_{\varphi_2}^{\varphi_1} C_{pB} \sin \varphi d\varphi$$

with  $C_p$  given by equation (2)

i.e.

$$C_Y = \int_{\varphi_1}^{\varphi_2} [(1 - 4 \sin^2 \varphi_0) - 8 \sin \varphi_0 \sin \varphi - 4 \sin^2 \varphi] \sin \varphi d\varphi + \int_{\varphi_2}^{\varphi_1} C_{pB} \sin \varphi d\varphi$$

$$= [1 - 4 \sin^2 \varphi_0 - C_{pB}](\cos \varphi_1 - \cos \varphi_2) - 4 \sin \varphi_0 [\cos \varphi_1 \sin \varphi_1 - \varphi_1$$

$$- \cos \varphi_2 \sin \varphi_2 + \varphi_2] - \frac{4}{3} [\cos \varphi_1 (\sin^2 \varphi_1 + 2) - \cos \varphi_2 (\sin^2 \varphi_2 + 2)]$$

$$(4)$$

The separation angle  $\varphi_1$  and  $\varphi_2$  are determined from an analysis of separation and from equation (3) respectively in terms of  $\varphi_0$ .

### Boundary Layer Separation

The separation angle  $\varphi_1$  is determined by integration of the boundary layer from the stagnation point  $\varphi = -\varphi_0$  to  $\varphi = \varphi_1$ .

The momentum equation is written as

$$U^2 \frac{d\bar{\theta}}{dx} + (2\bar{\theta} + \bar{\delta}^*)U \frac{dU}{dx} = \frac{\tau_0}{\rho} \quad (5)$$

where  $\bar{\theta}$  is the momentum thickness,  $\bar{\delta}^*$  is the displacement thickness and  $\tau_0$  is the shear stress at the wall. In dimensionless form for a cylinder this becomes

$$u^2 \frac{d\theta}{d\varphi} + (2\theta + \delta^*)u \frac{du}{d\varphi} = 2C_f \quad (6)$$

where  $\theta$  and  $\delta^*$  are dimensionless and  $C_f = \tau_0 / \frac{1}{2} \rho U^2$ .

Integration of equation (6) has been carried out in an approximate way<sup>2</sup> to give a solution for  $\theta$  as

<sup>1</sup>Separation on the rearward side is a function of circulation

<sup>2</sup>See Boundary Layer Theory by Hermann Schlichting.

$$\theta(u\theta)^{\frac{1}{n}} = C Re^{-\frac{1}{n}} u^{-d} \int_{-\phi_0}^{\phi} u^d d\phi$$

where  $Re$  is the Reynolds number.

This result applies to both laminar and turbulent boundary layers, if it is assumed that the boundary layer is wholly laminar or wholly turbulent from stagnation point to separation. For laminar flow  $n=1$ ,  $c=0.47$  and  $d=5$ , for turbulent flow  $n=4$ ,  $c=0.016$  and  $d=4^3$ . However, the condition for separation may also be written in terms of  $u$  and  $\theta$  as

$$\frac{\theta}{u} \left( -\frac{du}{d\phi} \right) (u\theta)^{\frac{1}{n}} = -\Gamma Re^{-\frac{1}{n}} \quad (8)$$

where  $-\Gamma=0.1567$  for laminar flow and  $-\Gamma=0.06$  for turbulent flow.

Elimination of  $\theta$  from equation (7) and (8) gives the condition for separation which must be satisfied by  $\phi_1$  i.e.,

$$\frac{-u'}{u^{\frac{d+1}{n}}} \int_{-\phi_0}^{\phi_1} u^d d\phi = \frac{-\Gamma}{c} \quad (9)$$

where  $u' = du/d\phi$  and  $u$  is given by equation (3).

In particular, for laminar flow  $-\frac{u'}{u^6} \int_{-\phi_0}^{\phi_1} u^5 d\phi = 0.334 \quad (9a)$

and for turbulent flow  $-\frac{u'}{u^5} \int_{-\phi_0}^{\phi_1} u^4 d\phi = 3.75 \quad (9b)$

Substitution for  $u$  gives the separation angle  $\phi_1$  in terms of  $\phi_0$ . This is plotted for laminar and turbulent flow in figure 2<sup>4</sup>.

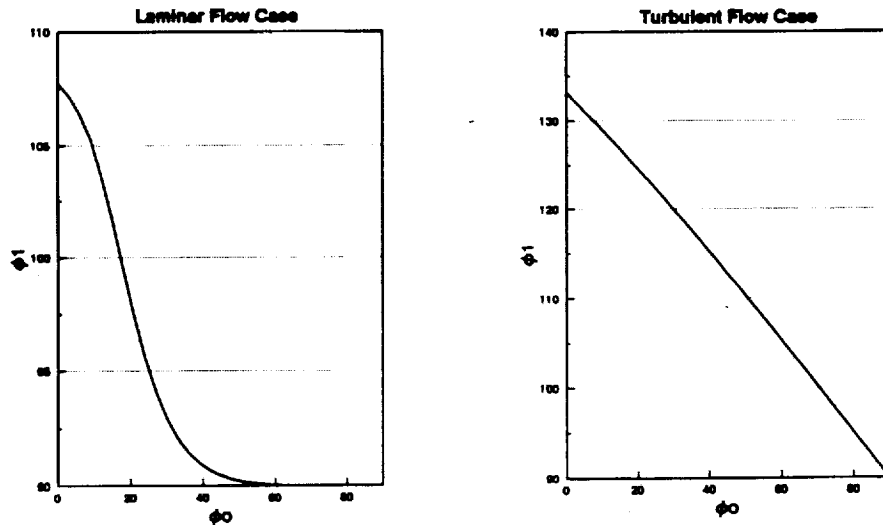
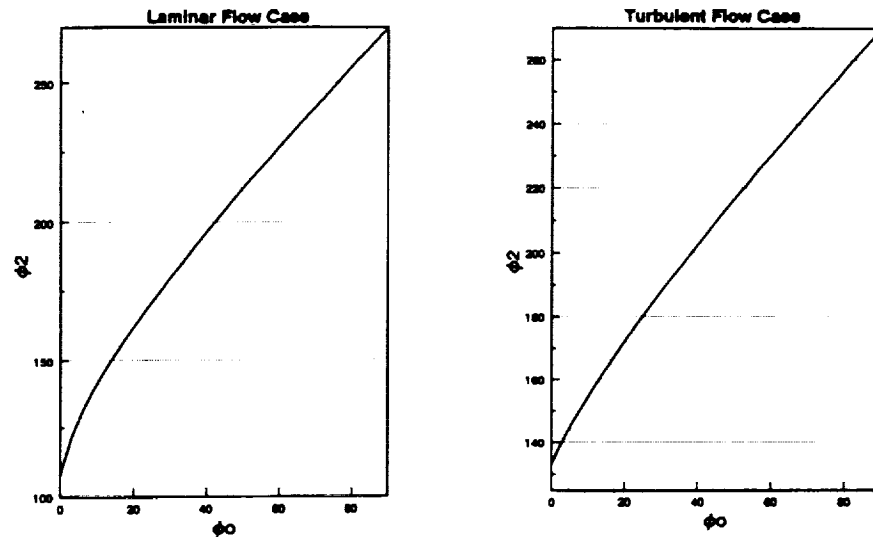


Figure 2. Pressure Side Separation Angle vs. Circulation Angle

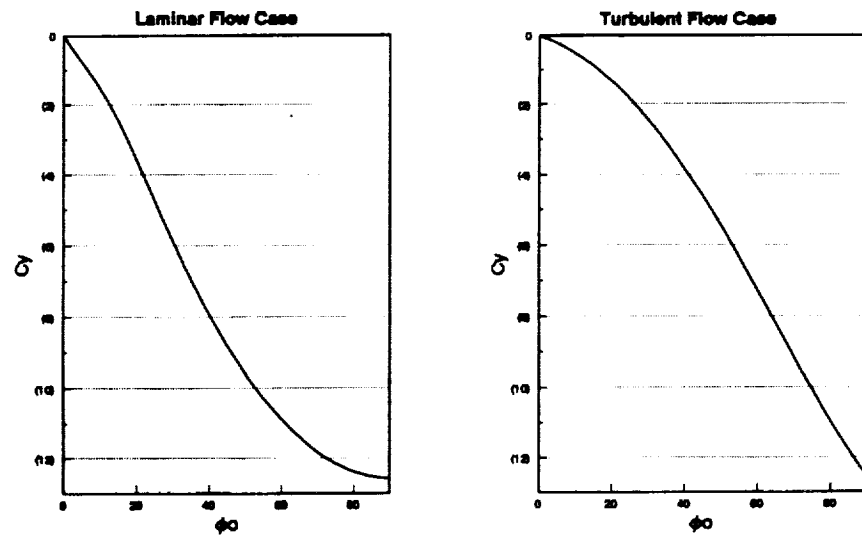
<sup>3</sup>Determined experimentally.

<sup>4</sup>Note that the results are independent of the Reynolds number

Use of these results permits  $\varphi_2$  to be calculated as a function of  $\varphi_0$  and therefore the side force coefficient can be found from equation (4). These are plotted in figures 3 and 4.



**Figure 3. Separation Angle with Suction vs. Circulation Angle**



**Figure 4. Side Force Coefficient vs. Circulation Angle**

### Required Normal Suction

In order to prevent separation at any location  $\varphi < \varphi_2$  on the suction side of the cylinder, it is necessary to introduce a normal suction distribution.

For a given value of  $\varphi_0$ , separation on the suction side will occur, in the absence of normal suction at a location  $\varphi_s$  found by integration of the boundary layer equation counterclockwise along the surface of the cylinder, i.e.  $\varphi_s$  is given by

$$\text{Laminar Flow : } \frac{u'}{u^6} \int_{-\varphi_0}^{\varphi_s} u^5 d\varphi = 0.334$$

or

$$\text{Turbulent Flow : } \frac{u'}{u^5} \int_{-\varphi_0}^{\varphi_s} u^4 d\varphi = 3.75$$

The values are shown as functions of  $\varphi_0$  in figure 5. This is the value of  $\varphi$  at which suction is introduced to prevent separation.

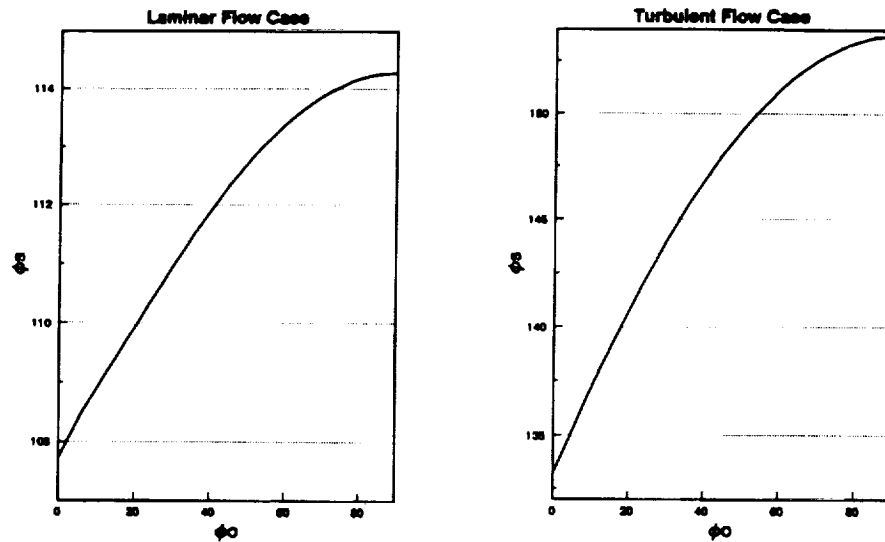


Figure 5. Suction Side Separation Angle vs. Circulation Angle

The momentum equation with suction is now written

$$u^2 \frac{d\theta}{d\varphi} + (2\theta + \delta)u \frac{du}{d\varphi} = 2C_f + v_0 u \quad (10)$$

where  $v_0 = V(\theta)/U$  is the velocity normal to the surface<sup>5</sup>. For  $C_f=0$ , the distribution of  $v_0$  required to cause the boundary layer to be continuous till the point of separation is

$$-v_0 = -u'\theta(2 + \frac{\delta}{\theta}) - u\theta' \quad (11)$$

However,  $\theta$  must satisfy

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<sup>5</sup> $v_0 < 0$  for normal suction.

$$\frac{\theta}{u} \left( -\frac{du}{d\phi} \right) (u\theta)^{1/2} = -\Gamma \text{Re}^{-1/2} \quad (8)$$

the condition for separation.

Elimination of  $\theta$  gives the suction distribution for  $\phi_2 < \phi < \phi_s$

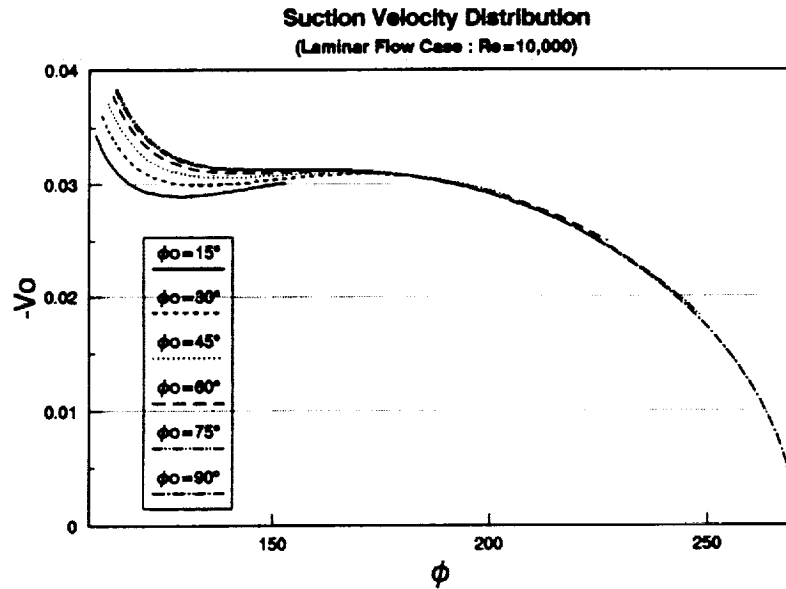
$$-v_0 = \text{Re}^{-1/2} (-\Gamma)^{1/2} (-u')^{1/2} \left( 2 + \frac{\delta}{\theta} \right) u^{n-1/2} \left[ 1 + \frac{n-1}{n+1} \frac{1}{2 + \delta/\theta} + \frac{n}{(n+1)(2 + \delta/\theta)} \frac{u(-u'')}{(-u')^2} \right] \quad (12)$$

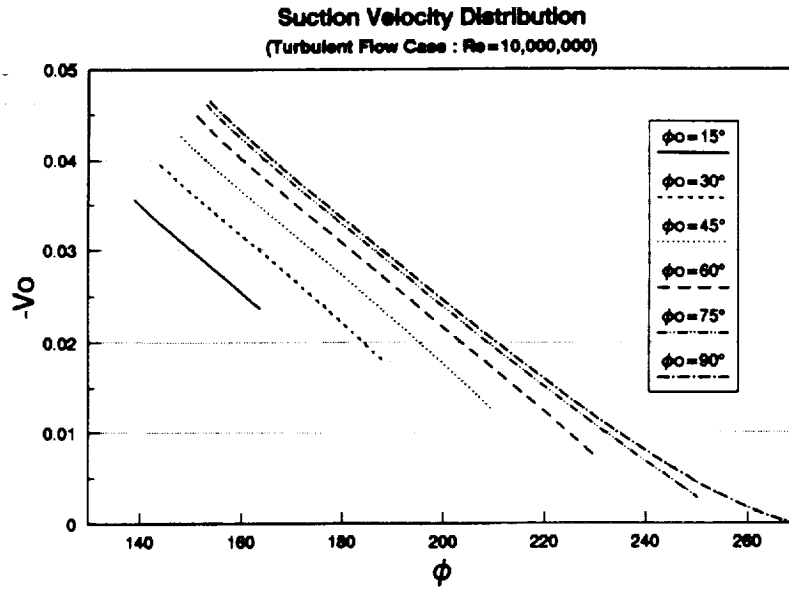
For laminar flow  $\delta/\theta = 3.5$  at separation whereas for turbulent flow  $\delta/\theta$  is taken as  $\delta/\theta = 2$ . With these values and the approximate values of  $-\Gamma$  and  $n$

$$\text{Laminar Flow : } -v_0 = \text{Re}^{-1/2} 2.18 (-u')^{1/2} \left[ 1 + 0.091 \frac{u(-u'')}{(-u')^2} \right] \quad (12a)$$

$$\text{Turbulent Flow : } -v_0 = \text{Re}^{-1/2} 0.484 (-u')^{1/2} u^{1/2} \left[ 1 + 0.174 \frac{u(-u'')}{(-u')^2} \right] \quad (12b)$$

The distribution of suction velocity between  $\phi = \phi_s$  and  $\phi = \phi_2$  for various values of  $\phi_0$  are shown in figure 6.



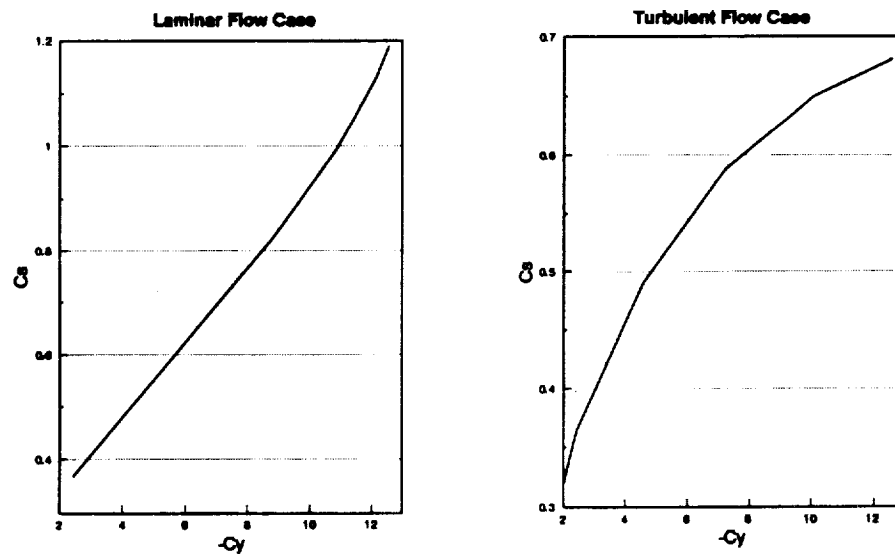


**Figure 6. Suction Velocity Distribution**

An integrated suction coefficient defined as

$$C_s = \left[ \frac{1}{2\pi} \int_{\phi_1}^{\phi_2} v_o d\phi \right] \left( \frac{Re}{10^6} \right)^{-\frac{1}{5}} \quad (13)$$

is shown as a function of the side force coefficient in figure 7.



**Figure 7. Suction Coefficient vs. Side Force Coefficient**

## **Conclusion**

The relations between the circulation angle, side force coefficient and required suction velocity were inspected. The distribution of suction obtained was the most effective arrangement to prevent separation and give a corresponding side force. However, the magnitude of suction varies and range over which it is applied is a function of the separation angle  $\varphi_0$ . Therefore, implementing distributed suction in actual situations is very difficult and a method to obtain a constant magnitude of suction velocity for an arbitrary fixed range must be sought. This problem requires further investigation. Nonetheless, distributed suction is a favorable method to generate side force and can be implemented for control devices for fighter aircraft at very high angles of attack.



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